A CELL-BASED SMOOTHED FINITE ELEMENT USING A FOUR-NODE QUADRILATERAL ELEMENTS FOR STATIC AND EIGEN BUCKLING ANALYSIS OF NON-WOVEN FABRIC

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This paper presents the modification of curvature and transverse shear strains of a four-node quadrilateral Reissner-Mindlin plate element (Q4) using the cell-based smoothed finite-element method (CS-FEM) for static and eigen buckling analysis of non-woven fabric. This is to improve the computation efficiency and accuracy of the Q4 element without much modifying to the configurations of Q4 in FEM. The Q4 plate elements exists the phenomena, known as shear locking, that is caused by parasitic shear deformation energy which yields an artificial additional stiffness, appear as the bending plates becomes progressively thinner. In order to overcome this drawback, the reduced integration methods on smoothing domains of Q4 element was used. The non-woven fabric material was assumed as isotropic material, and its mechanical properties was measured using the Kawabata's evaluation system for fabric (KES-FB). The numerical implementation and results demonstrated that the Q4 plate element based on CS-FEM possesses the following improved properties: accurate than those of the FEM using quadrilateral elements having the same sets of nodes; this method can be also temporally stable for buckling analysis, and the computational efficiency is better than the FEM using the same configurations of nodes.

Key Words: Reissner–Mindlin plate theory, cell-based smoothed strain technique, plain-woven fabric, Kawabata's evaluation system for fabric

1. INTRODUCTION AND FINITE ELEMENT FORMULATION

This work presents a four-node quadrilateral bending plate element (Q4) based on the Reissner-Mindlin plate theory [1, 2] and a strain smoothing method in finite elements for eigenvalue analysis of non-woven fabric. The review of eigenvalue analysis using the thin plate/shell finite element models can be referred to [3], and the different stages of development of plate/shell finite elements can be found in the works of Yang and coworkers [4].

Now, let the reference domain Ω be discretized into N_e four-node quadrilateral finite elements such that $\Omega = \bigcup_{i=1}^{N_e} \Omega_i^e \,, \Omega_i^e \cap \Omega_j^e = \emptyset,\ i \neq j$, where Ω_i^e and Γ_i^e denote the domain of the ith generic element and its boundary, respectively. The finite element solution $\boldsymbol{u}^h = (w^h, \beta_x^h, \beta_y^h)^T$ of a displacement model for the Mindlin/Reissner is approximated by

$$\boldsymbol{u}^h = \sum_{I=1}^{4} \boldsymbol{N}_I(\mathbf{x})\boldsymbol{d}_I = \sum_{I=1}^{N_n} \begin{bmatrix} N_I(x) & 0 & 0 \\ 0 & N_I(x) & 0 \\ 0 & 0 & N_I(x) \end{bmatrix} \boldsymbol{d}_I, \tag{1}$$

where the nodal shape functions $N_I(x)$ are constants in the CS-FEM model [5], and $\boldsymbol{d}_I = [w_I \quad \beta_{xI} \quad \beta_{yI}]^T$ is the vector of nodal degrees of freedom of $\boldsymbol{u}^h$ associated with node I.

Considering the variational principles and the weak form for the static and buckling analysis, the discretized governing equation can be expressed as

$$\left[\left(\int_\Omega \boldsymbol{B}^{\mathrm{b}^T}\boldsymbol{c}^{\mathrm{b}}\boldsymbol{B}^{\mathrm{b}} d\Omega + \int_\Omega \boldsymbol{B}^{\mathrm{s}^T}\boldsymbol{c}^{\mathrm{s}}\boldsymbol{B}^{\mathrm{s}} d\Omega\right) - \lambda_{cr}\int_\Omega \boldsymbol{B}^{\mathrm{G}^T}\boldsymbol{\tau}\boldsymbol{B}^{\mathrm{G}} d\Omega\right]\boldsymbol{d} = \left((\boldsymbol{K}^{\mathrm{b}}+\boldsymbol{K}^{\mathrm{s}}) - \lambda_{cr}\boldsymbol{K}_{\mathrm{G}}\right)\boldsymbol{d} = 0, \quad (2)$$

where, $\lambda_{cr}$is the critical buckling load. Matrices $\boldsymbol{K}^{\mathrm{b}}$, $\boldsymbol{K}^{\mathrm{s}}$ and $\boldsymbol{K}_{\mathrm{G}}$ are, respectively, the bending stiffness, the shear stiffness and geometrical stiffness of the structure (or global), while $\boldsymbol{c}^{\mathrm{b}}$, $\boldsymbol{c}^{\mathrm{s}}$ and $\boldsymbol{\tau}$ are constant matrices related to material properties and the in-plane pre-buckling stresses. The strain matrices associate with each node I of the element Ω^e in respect of $\boldsymbol{B}^{\mathrm{b}}$, $\boldsymbol{B}^{\mathrm{s}}$and $\boldsymbol{B}^{\mathrm{G}}$are

$$\boldsymbol{B}_I^{\mathrm{b}} = \begin{bmatrix} 0 & N_{I,x} & 0 \\ 0 & 0 & N_{I,y} \\ 0 & N_{I,y} & N_{I,x} \end{bmatrix}, \boldsymbol{B}_I^{\mathrm{s}} = \begin{bmatrix} N_{I,x} & N_I & 0 \\ N_{I,y} & 0 & N_I \end{bmatrix}, \boldsymbol{B}_I^{\mathrm{G}} = \begin{bmatrix} N_{I,x} & 0 & 0 \\ N_{I,y} & 0 & 0 \\ 0 & N_{I,x} & 0 \\ 0 & N_{I,y} & 0 \\ 0 & 0 & N_{I,x} \\ 0 & 0 & N_{I,y} \end{bmatrix}. \quad (3)$$

The smoothing operation [6, 7] for cell-based model performed over the kth smoothing domain $\Omega_k^s \subset \Omega^e$ is addressed as:

$$\bar{\nabla}\boldsymbol{u}(\mathrm{x}_k) = \int_{\Omega_i^s} \nabla\boldsymbol{u}(\mathrm{x})\Phi(\mathrm{x}-\mathrm{x}_k)d\Omega\,, \Phi \geq 0 \text{ and } \int_{\Omega_i^s} \Phi d\Omega = 1, \Phi(\mathrm{x}-\mathrm{x}_k) = \begin{cases} \dfrac{1}{A_k^s}, & \mathrm{x} \in \Omega_k^s \\ 0, & \mathrm{x} \notin \Omega_k^s \end{cases} \quad (4)$$

Considering the smoothing function Φ and its condition in Eq. (4), the smoothed gradient of displacement can be defined as

$$\bar{\boldsymbol{\varepsilon}}(\mathrm{x}_k) = \int_{\Omega_k^s} \nabla\boldsymbol{u}(\mathrm{x}).\Phi(\mathrm{x}-\mathrm{x}_k)d\Omega = \frac{1}{A_k^s}\int_{\Gamma_k^s} \boldsymbol{n}(\mathrm{x}).\boldsymbol{u}(\mathrm{x})d\Gamma, \quad (5)$$

where $\boldsymbol{u}(\mathrm{x})$ is the generalized displacement vector and $\boldsymbol{n}(\mathrm{x})$ is the outward unit normal matrix containing the components of the outward unit normal vector to the boundary of kth cell $\Gamma_k^s$of $\Omega_k^s \subset \Omega^e$, and $A_k^s = \int_{\Omega_k^s} d\Omega$ is the area of sub-cell. Applying the Eq. (4) to the Eq. (3) in order to evaluate the corresponding trains fields.

2. NUMERICAL IMPLEMENTATION, RESULTS AND CONCLUSIONS

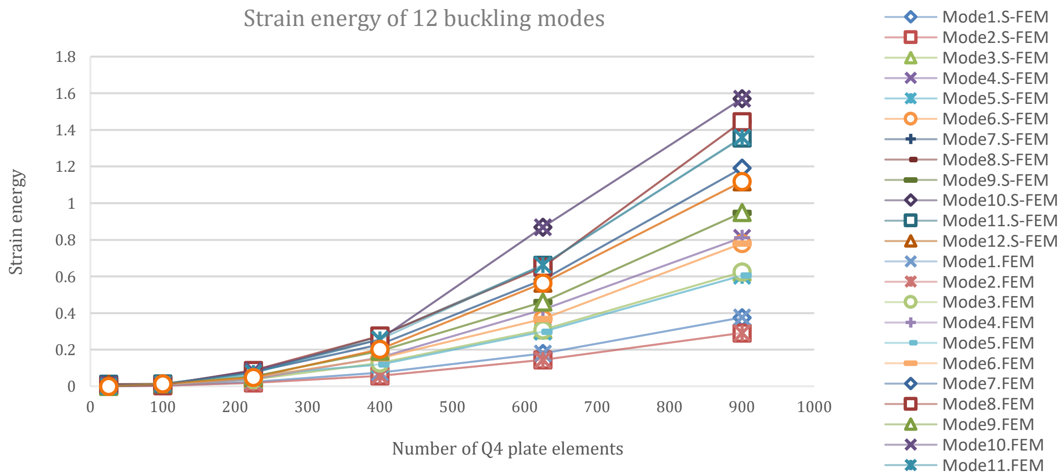

Figure 1. The strain energy of 12 buckling modes between the implemented CS-FEM model and FEM for clamped edge boundary conditions with different mesh density.

The twelve modes of the critical buckling load λ of a square fabric sheet was assumed as an isotropic material and clamped edges boundary condition was carried out by solving the linearized eigenvalue problem developed in the previous section, as shown in the Fig. 1.

Mechanical parameters of non-woven fabric sample for numerical implementation were measured with Kawabata evaluation system for fabrics (KES-FB) [8, 9] and derived as: elastic modulus [gf/cm], $E = 3823.7993$, Poisson's ratio $v = 0.0778$. The numerical programming was implemented using Python and NumPy which is the fundamental package for scientific computing with Python.The clamped edges condition was applied for meshes (elements) including 5x5, 10x10, 15x15 and 20x20.

The eigenvalues of 12 eigen-modes extracted from buckling analysis computed by the implementation of CS-FEM model was approximate and coincided with that one of FEM on the same boundary conditions and mesh configurations. The linear shape functions in S-FEM model are constants, but the corresponding billinear shape functions of a Q4 element in FEM are interpolated in natural coordinates (ξ, η, ζ). Thus, the strain smoothing technique reduces the numerical implementation and computation time in terms of the central processing unit time, present trends of numerical mechanics in modeling and simulations regarding to more complex mathematical models in textile engineering and sciences.

ACKNOWLEDGMENTS

The authors wish to express their acknowledgment to "Project UID/CTM/00264/2019 of 2C2T – Centro de Ciência e Tecnologia Têxtil, funded by National Founds through FCT/MCTES", and also to the FCT funding – Foundation for Science and Technology within the scope of the project "PEST UID/CTM/00264; POCI-01-0145-FEDER-007136".

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