A CELL-BASED SMOOTHED DISCRETE SHEAR GAP USING TRIANGULAR ELEMENTS FOR FREE VIBRATION ANALYSES OF PLAIN-WOVEN FABRIC

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A cell-based smoothed discrete shear gap method for free vibration analysis of plain-woven fabric is formulated by incorporating the cell-based strain smoothing technique with the discrete shear gap method using three-node triangular Reissner–Mindlin plate element. In this numerical work, the domain of each element will be sub-divided into three triangle domains by connecting the three field nodes to the central point of the element. Then the strain smoothing technique is used to evaluate the curvature strains on these sub-domains. To overcome the shear locking, the discrete shear gap method is applied The fabric material was assumed as an orthotropic material, and its mechanical properties was measured using the Kawabata's evaluation system for fabric (KES-FB). The numerical result using formulated plate element is, therefore, free of shear locking, good accuracy, and conditionally stable but effective performance in terms of CPU time compared to the other plate elements in finite-element methods.

Key Words: Reissner–Mindlin plate theory, cell-based smoothed discrete shear gap technique, plain-woven fabric, Kawabata's evaluation system for fabric

1. INTRODUCTION AND FINITE ELEMENT FORMULATION

This work presents a study of three-node triangular plate bending element (T4) based on the Reissner-Mindlin plate theory [1] and a strain smoothing method in finite elements for free vibration analysis of plain-woven fabric. Reader can refer to [2] for reviewing of eigenvalue analysis using the thin plate/shell finite element models. The works of Yang and coworkers [3] presented the different stages of development of plate/shell finite elements.

For the free vibration analysis of plain-woven fabric using Reissner-Mindlin plates, considering the standard Galerkin weak form and the dynamic form of energy principle , which give

$$\int_{\Omega} \delta\boldsymbol{\kappa}^T \boldsymbol{c}^{\mathrm{b}} \boldsymbol{\kappa} d\Omega + \int_{\Omega} \delta\boldsymbol{\gamma}^T \boldsymbol{c}^{\mathrm{s}} \boldsymbol{\gamma} d\Omega + \int_{\Omega} \delta\mathbf{u}^T \boldsymbol{m}\ddot{\mathbf{u}} d\Omega = 0 \tag{1}$$

where $\boldsymbol{c}^{\mathrm{b}}$and $\boldsymbol{c}^{\mathrm{s}}$ are matrices related to bending deformation and shear deformation, $\boldsymbol{\kappa}$, $\boldsymbol{\gamma}$ and $\boldsymbol{m}$ are curvature of the deflected plate, the shear strains and the mass matrix, respectively, which can be found in [4]. Now, let the reference domain Ω be discretized into N_e three-node triangular plate bending elements such that $\Omega = \bigcup_{i=1}^{N_e} \Omega_i^e\,, \Omega_i^e \cap \Omega_j^e = \emptyset,\, i \neq j$, where Ω_i^e and Γ_i^e denote the domain of the ith generic element and its boundary, respectively. The finite element solution $\boldsymbol{u}^h = (w^h, \beta_x^h, \beta_y^h)^T$ of a displacement model for the Mindlin/Reissner is approximated by

$$\boldsymbol{u}^h = \sum_{I=1}^{3} \boldsymbol{N}_I(\mathbf{x}) \boldsymbol{d}_I = \sum_{I=1}^{N_n} \begin{bmatrix} N_I(x) & 0 & 0 \\ 0 & N_I(x) & 0 \\ 0 & 0 & N_I(x) \end{bmatrix} \boldsymbol{d}_I, \tag{2}$$

where the nodal shape functions $N_I(x)$ are constants in the CS-FEM model [5], and $\boldsymbol{d}_I = [w_I \quad \beta_{xI} \quad \beta_{yI}]^T$ is the vector of nodal degrees of freedom of $\boldsymbol{u}^h$ associated with node I.

Applying FEM procedure regarding to the Eq. (1 and 2) for free vibration analysis yields

$$(\boldsymbol{K}^{\mathrm{b}} + \boldsymbol{K}^{\mathrm{s}} - \omega^2 \boldsymbol{M})\boldsymbol{d} = \boldsymbol{0}, \quad (3)$$

where $\boldsymbol{K}^{\mathrm{b}}$, $\boldsymbol{K}^{\mathrm{s}}$ and $\boldsymbol{M}$ are the bending stiffness, the shear stiffness and mass matrix of the structure (or global), respectively. While ω is the natural frequency.

The smoothing operation [6, 7] for cell-based model performed over the kth smoothing domain $\Omega_k^s \subset \Omega^e$ is addressed as:

$$\overline{\nabla}\boldsymbol{u}(\mathrm{x}_k) = \int_{\Omega_i^s} \nabla\boldsymbol{u}(\mathrm{x})\Phi(\mathrm{x}-\mathrm{x}_k)d\Omega, \Phi \geq 0 \text{ and } \int_{\Omega_i^s} \Phi d\Omega = 1, \Phi(\mathrm{x}-\mathrm{x}_k) = \begin{cases} \frac{1}{A_k^s}, & \mathrm{x} \in \Omega_k^s \\ 0, & \mathrm{x} \notin \Omega_k^s \end{cases} \quad (4)$$

Considering the smoothing function Φ and its condition in Eq. (4), the smoothed gradient of displacement can be defined as

$$\bar{\boldsymbol{\varepsilon}}(\mathrm{x}_k) = \int_{\Omega_k^s} \nabla\boldsymbol{u}(\mathrm{x}).\Phi(\mathrm{x}-\mathrm{x}_k)d\Omega = \frac{1}{A_k^s}\int_{\Gamma_k^s} \boldsymbol{n}(\mathrm{x}).\boldsymbol{u}(\mathrm{x})d\Gamma, \quad (5)$$

where $\boldsymbol{u}(\mathrm{x})$ is the generalized displacement vector and $\boldsymbol{n}(\mathrm{x})$ is the outward unit normal matrix containing the components of the outward unit normal vector to the boundary of kth cell $\Gamma_k^s$of $\Omega_k^s \subset \Omega^e$, and $A_k^s = \int_{\Omega_k^s} d\Omega$ is the area of sub-cell. Applying the Eq. (4) to the curvature of the deflected plate $\boldsymbol{\kappa}$ and the shear strains $\boldsymbol{\gamma}$ in order to evaluate the corresponding trains fields.

For combining the cell-based strain smoothing technique and the discrete shear gap method (DSG) whose element is shear-locking-free, as presented in Ref. [8].

2. NUMERICAL IMPLEMENTATION, RESULTS AND CONCLUSIONS

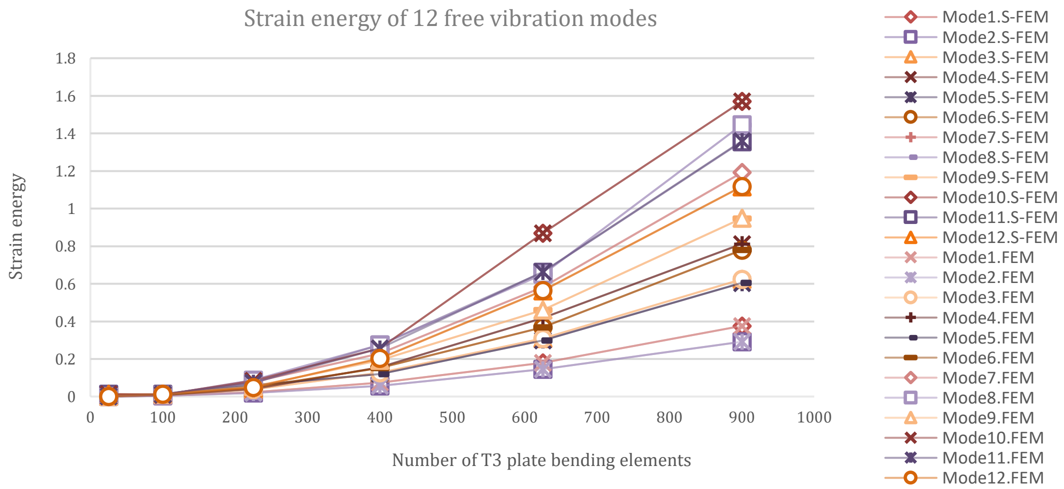

Figure 1. The strain energy of 12 free vibration modes between the implemented CS-FEM model and FEM for clamped edge boundary conditions with different mesh density.

Mechanical parameters of plain-woven fabric sample for numerical implementation were measured with KES-FB [9, 10] and derived as: elastic modulus [gf/cm], $E = 3823.7993$, Poisson's ratio $\upsilon = 0.0778$. The numerical programming was implemented using Python and NumPy which is the fundamental package for scientific computing with Python.The clamped edges condition was applied for meshes (elements) including 5x5, 10x10, 15x15 and 20x20.

The eigenvalues of 12 eigen-modes extracted from free vibration analysis computed by the implementation of CS-FEM model and DSG was approximate and coincided with that one of FEM on the same boundary conditions and mesh configurations. The formulated element was also free of shear locking, good accuracy, and conditionally stable but effective performance in terms of CPU time compared to the other plate elements in finite-element methods. Thus the combination of S-FEM and DSG trend to numerical mechanics in modeling and simulations regarding to more complex mathematical models in textile engineering and sciences.

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